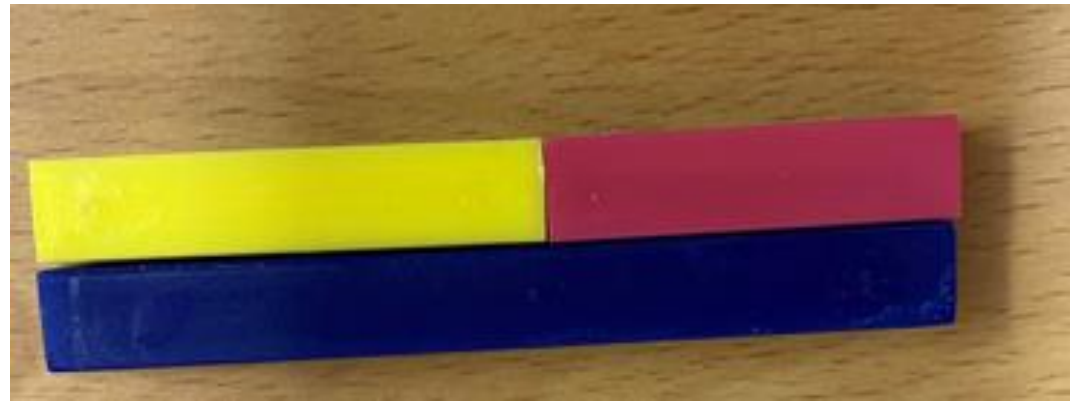
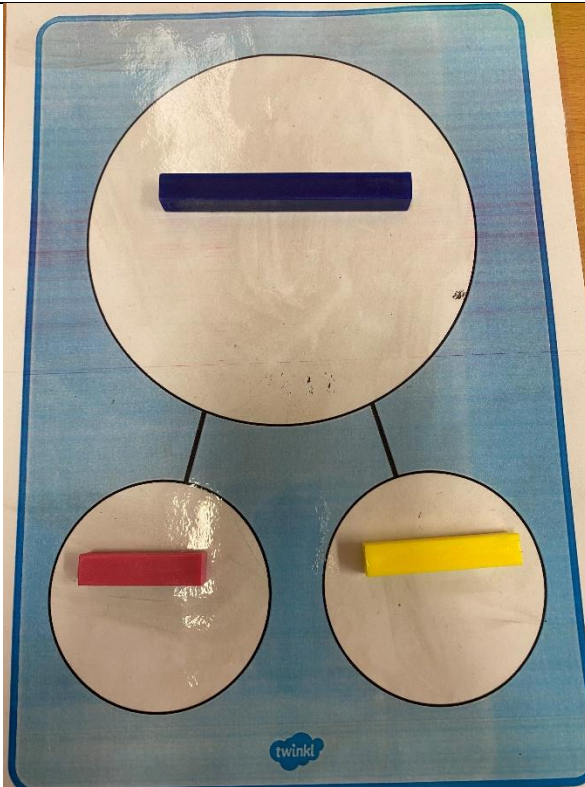


Addition - No Numbers

Main Year
Groups

Children should build their vocabulary with Cuisenaire Rods. The part-whole method and the bar model should be introduced with Cuisenaire rods. The rods should not be given numerical values. NB – check for signs of colour deficiency/blindness.

R



Addition

Combining sets of objects

Pre-requisites and explicit pre-practice:

1-1 Correspondence

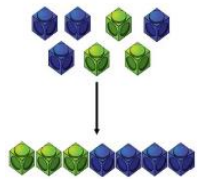
Counting of objects

Dividing objects into two groups

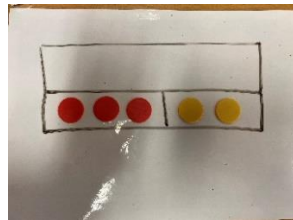
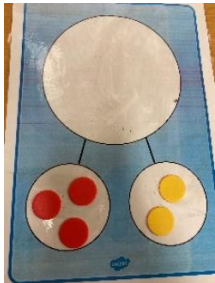
Understand that the last number counted is the amount in the set.

Main Year
Groups

Concrete

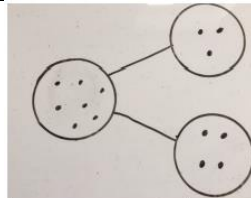


Combine a variety of objects to make a whole.



Most children should do this step by the end of R.

Pictorial



Children use their own efficient markings to represent the concrete objects.



Most children should do this abstract step by the end of Y1

Abstract

$$4 + 3 = 7$$

7	
4	3

Most children should do this abstract step by the end of Y1

Holding a number and counting on.

Pre-requisites and explicit pre-practice:

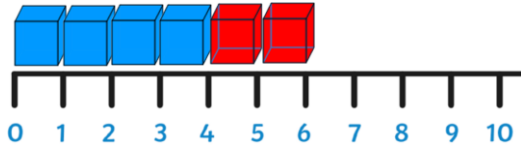
Familiarisation with the number line, looking at patterns, recognising numbers etc.

Oral practice at counting on from a given number

Number recognition

'Holding' a number and understanding that you don't need to start from zero each time.

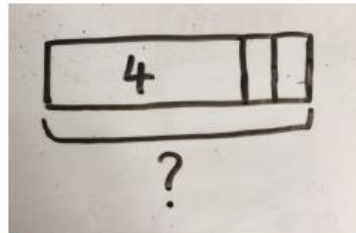
R, 1, 2



Children use practical objects with a number line. A bead string will also help.

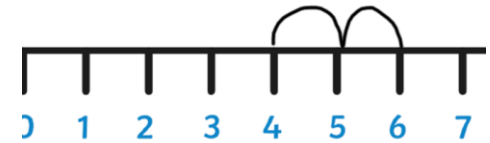


Most children should do this by the end of R,



A bar model that encourages children to count on rather than count all.

Most children should do this by the end of Y1



$$4 + 2 = 6$$

Abstract number line.

What is 2 more than 4?

What is 4 plus 2?

What is the sum of 4 and 2?

Most children should do this with numbers <20 and with counting back a few steps by the end of R

Most children should do this by the end of Y1

Partition and bridge

Pre-requisites and explicit pre-practice:

Instant recognition of amounts on a tens frame

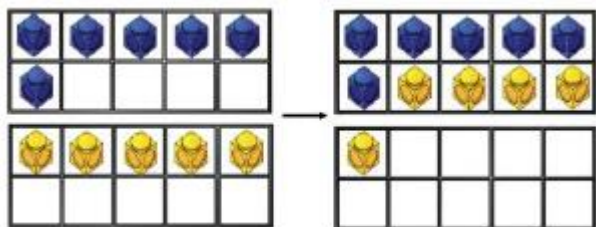
Understanding of part-whole

Recall of bonds to 10 –VITAL!

Understanding and fluency with teens numbers as 10 and 'some' more.

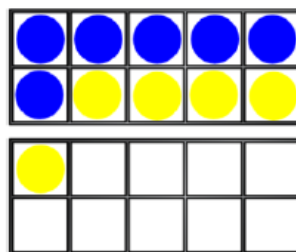
Conversations and exploration about why certain partitions are needed. E. G In the calculation below ($6 + 5 = 11$), why would partitioning the 5 into a 3 and a 2 not be helpful? This needs to be done LOTS!

1, 2, 3



Children begin to partition and bridge by using the tens frames

Most children should do this mentally by the end of Y1. It is vital that children know their bonds to 10 before they can master this.



Children use their own marks to draw in a tens frames. They also then sketch their own tens frames.

$$6 + 5 = 11$$

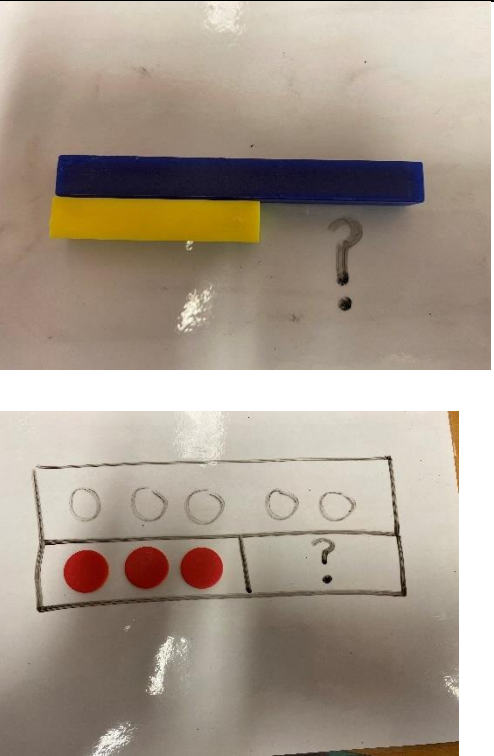
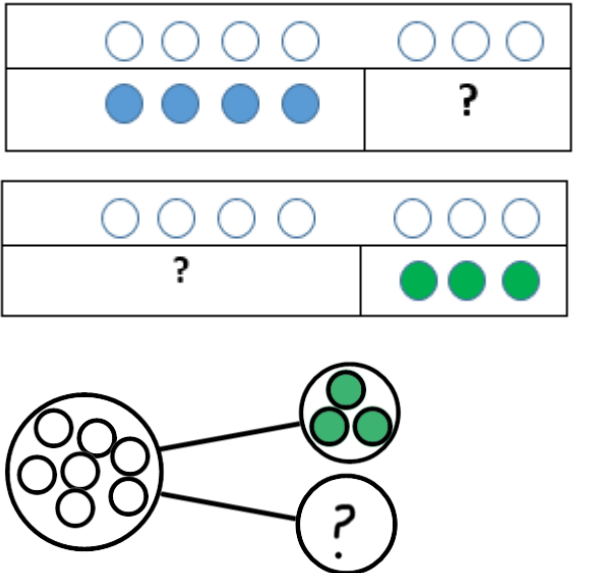
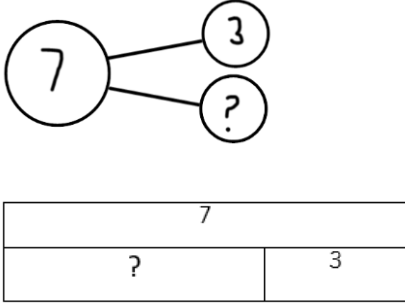
11		
6	5	
6	4	1

Children make jottings onto abstract calculations before partitioning and bridging mentally.

Most children should do this with written scaffold by the end of Y2

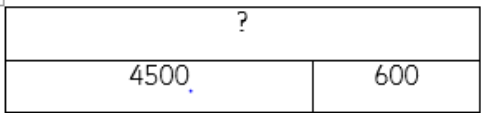
Most children should do this mentally by the end of Y3

Use of part-whole and bar model in KS1 for 'missing number' problems.

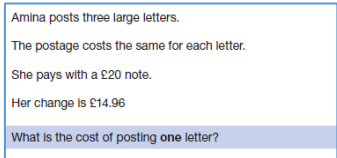
Main Year Groups	Concrete	Pictorial	Abstract
1, 2			<p> $4 + \square = 7$ $\square + 3 = 7$ $\square = 4 + 3$ $7 = \square + 3$ $7 = 4 + \square$ </p>  <p><i>Most pupils should be able to solve missing number problems by the end of Y1</i></p>

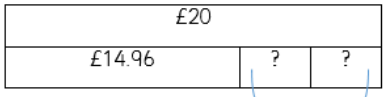
Children should continue to use the bar model (or part whole) through to Y6 to solve missing number problems in a variety of situations:


$7 \left| \begin{array}{|c|} \hline \square \\ \hline \end{array} \right. = 4,500 + 600$




or









Adding O to TO and TO to TO (No crossing boundaries)

Pre-requisites and explicit pre-practice:

- Counting in tens
- Mental addition of multiples of ten
- Mental addition/recall of adding one-digit numbers to one-digit numbers
- Partitioning numbers into tens and ones
- Representing numbers with arrow cards, dienes and place value counters.

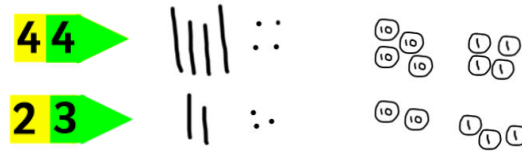
2,3

44
28

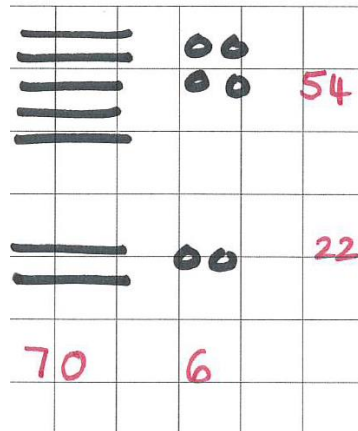


$44 + 28 =$
Children to use Dienes to add two 2-digit numbers by combining tens and ones. Place value arrow cards can be used for partitioning.

Most children should do this by the end of Y2



$$54 + 22 = \left[\right]$$



Children represent their calculations with drawings. They begin to organise them vertically.
Most children should do this by the end of Y2

Tens	Ones
40	4
20	3
60	7

$$44 + 23 = 67$$

or

$$\begin{array}{r} 40 \quad 4 + \\ 20 \quad 3 \\ \hline 60 \quad 7 \end{array}$$

?	
44	23

Children represent their calculations horizontally and vertically.

Most children should do this abstract step by the end of Y3. (This can be with the use of the place value frame as a scaffold.)

Adding TO to TO (Crossing boundaries – step 1)

Pre requisites and explicit pre-practice:

Exchanging ones for tens with Dienes and place value counters

Understand that ten ones are the same as one ten.

Mentally adding a number between 11 and 19 inclusive to a multiple of 10.

3,4

As above

$$44 + 28 =$$

Children to use Dienes and place value counters to add two 2-digit numbers by combining tens and ones. Place value arrow cards can be used for partitioning.

As above

Children represent their calculations with drawings. They organise them vertically.

$$\begin{array}{r} 40 + 4 + \\ 20 + 8 \\ \hline 60 + 12 = 72 \end{array}$$

?	
44	28

Children represent their calculations horizontally and vertically.

Most children should do this abstract step by the end of Y3. (This can be with the use of the frame as a scaffold.)

Adding TO to TO (Crossing boundaries – step 2)

Pre-requisites and explicit pre-practice:

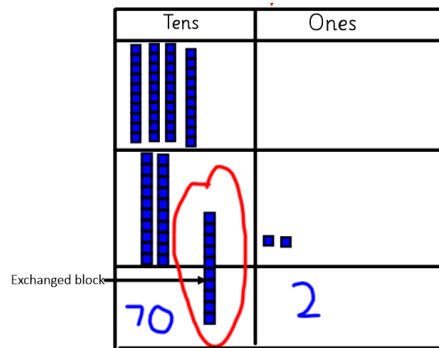
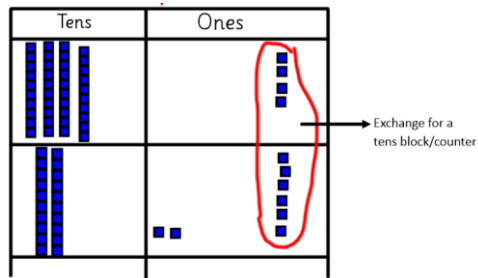
Exchanging ones for tens with Dienes and place value counters

Understand that ten ones are the same as one ten.

Mentally adding a number between 11 and 19 inclusive to a multiple of 10.

3,4

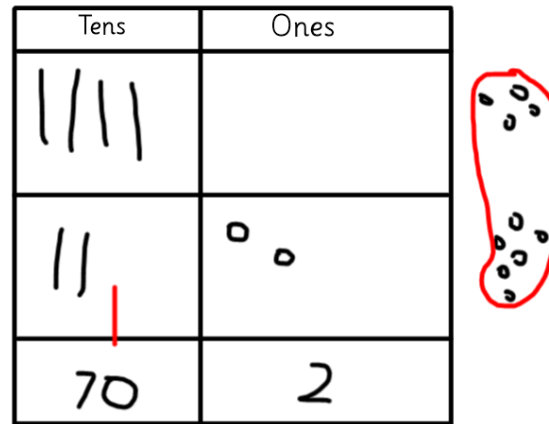
Using the frames as for no boundaries.



$44 + 28 =$

Children to use Dienes and place value counters to add two 2-digit numbers by combining tens and ones. They exchange the counters/blocks when the ones column has more than 9 in total.

Most children should do this abstract step by the end of Y3. (This can be with the use of the frame as a scaffold.)



Children represent their calculations with drawings. They organise them vertically.

Most children should do this abstract step by the end of Y3. (This can be with the use of the frame as a scaffold.)

$$\begin{array}{r} 44 + \\ 28 \\ \hline 72 \\ \hline 1 \end{array}$$

?	
44	28

Children represent their calculations vertically. The bar model will help develop understanding of inverse operations.

Most children should do this abstract step by the end of Y4. (This can be with the use of the frame as a scaffold.)

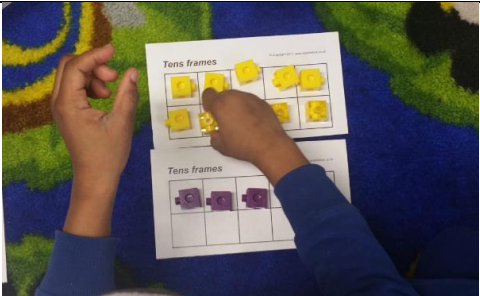
Subtraction

Subtracting Ones

Pre-requisites and explicit pre-practice:
1-1 Correspondence
Counting of objects
Number names
Understanding of more/fewer
Understand that the last number counted is the amount in a set

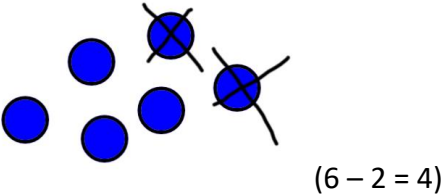
Main Year
Groups

Concrete



Most children should do this by the end of R.

Pictorial



Most children should do this by the end of R.

Abstract

$$6 - 2 = 4$$

Most children should do this by the end of Y1

Counting Back

Pre-requisites and explicit pre-practice:

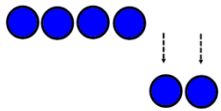
Number names

Count backwards from 10

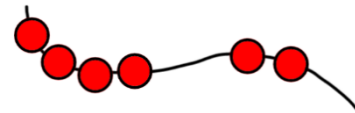
1-1 correspondence

(Children should continue to use counting back when introduced to fractions, decimals and negative numbers in years 4-6)

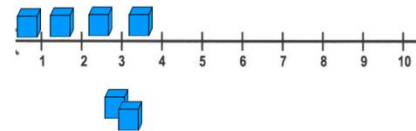
R, 1, 2



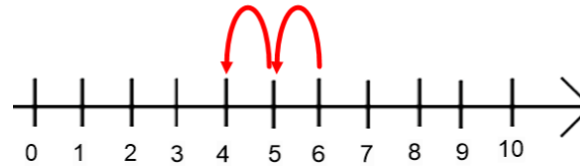
Child knows how many to start with.
As they remove each object, they
count backwards. "6, 5, 4...")



Bead string can be used
when counting
backwards.



Objects can be placed on a number line and then taken away.



"Put six in your head and count back two. What
number are you on?"

$$6 - 2 = 4$$

*Most children should do this with numbers <20 and with
counting back a few steps by the end of R*

Most children should do this by the end of Y1

Finding the difference

Pre-requisites and explicit pre-practice:

Number names

Count backwards from 10

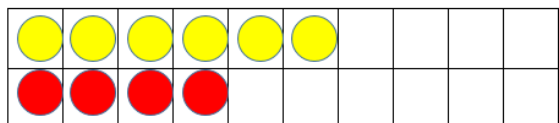
1-1 correspondence

(Children should continue to use counting back when introduced to fractions, decimals and negative numbers in years 4-6)

1, 2, 3

Most children should be able to describe the difference between two towers of cubes by the end of R.

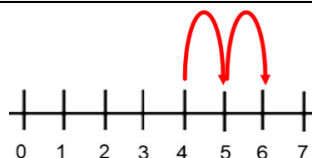
Objects can be laid on a bar model.



{
}
{
}

'Sameness'
Difference

4
?



(What is the difference between 4 and 6?
 $6 - 4 = ?$)

Most children should be able to do this by the end of Y1.

"Art has 4 sweets and Bart has 6 sweets. How many more does Bart have?"

$$6 - 4 = 2$$

**IMPORTANT* Children should be taught that finding the difference is not always an efficient strategy. For example, $103 - 4 = ?$ It would be more efficient to count back)*

Most children should be able to do this by the end of Y1.

Partition and Bridge

Pre-requisites and explicit pre-practice:

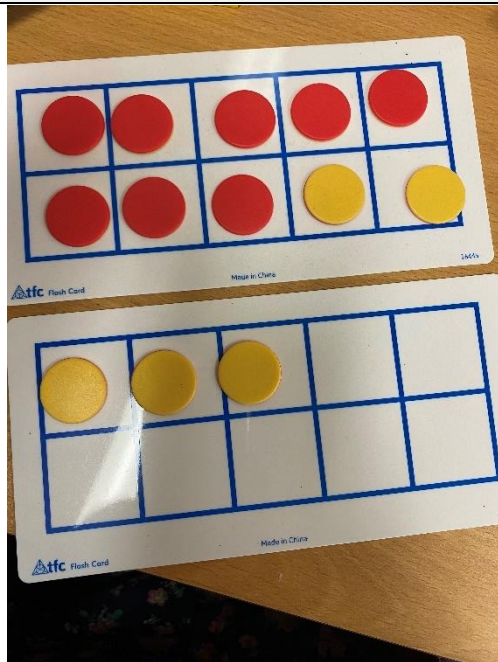
Number names

Count backwards from 10

1-1 correspondence

(Children should continue to use counting back when introduced to fractions, decimals and negative numbers in years 4-6)

1, 2, 3



Most children should do this by the end of Y1

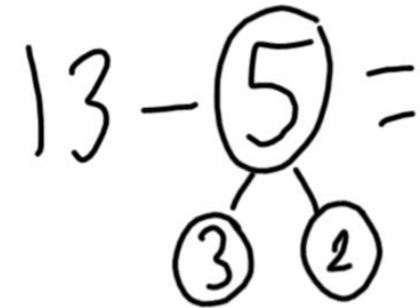


$(13 - 5 = ?)$

Subtract 3

Subtract 2

Most children should do this by the end of Y1



Most children should do this by the end of Y2

Most children should do this mentally by the end of Y3

Regroup Tens and Ones

Pre-requisites and explicit pre-practice:

Number names

Count backwards from 10

1-1 correspondence

Swapping/exchanging ones blocks for tens blocks

2, 3



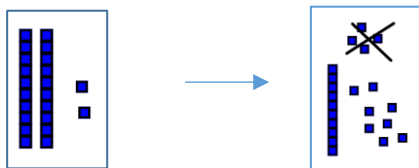
Before calculating, children need to practise JUST exchanging.

Most children should be able to do simple exchange without calculation, by the end of Y1.

Then...

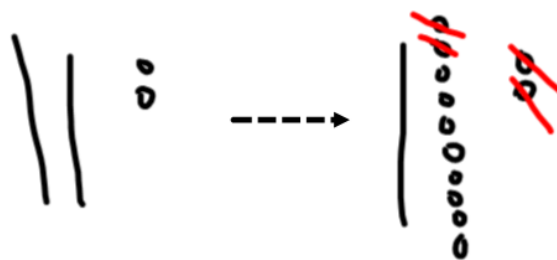
$$22 - 4$$

(One of the tens needs to be exchanged for ones)



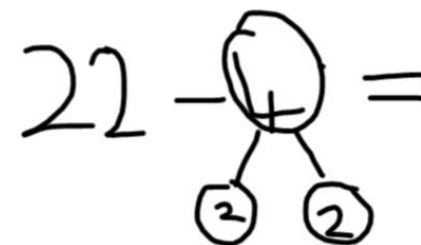
Most children should do this mentally by the end of Y2

$$22 - 4 =$$



22	
4	18

Most children should do this mentally by the end of Y2



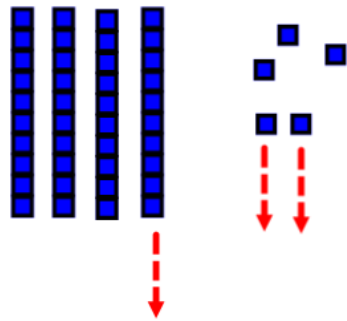
Most children should do this by the end of Y2

22	
4	18

Most children should do this mentally by the end of Y2

Column Subtraction (No Regrouping)

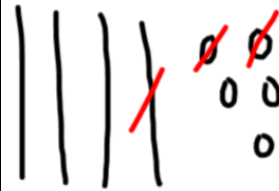
2, 3



$45 - 12 =$

Dienes should be placed onto place value grids to emphasise their value.

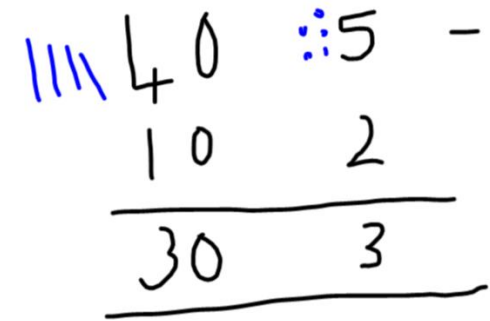
Most children should do this by the end of Y2



$45 - 12 =$

45	
12	33

Most children should do this by the end of Y2



45	
12	33

Most children should do this by the end of Y3

Column Subtraction (With Regrouping) – Step 1

3

$$53 - 24 =$$

Tens	Ones

Subtract 20

Exchange a ten for ones.

Place Value Counters should also be used when children are secure with place value.

Tens	Ones

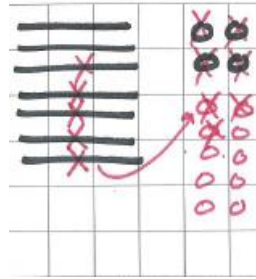
Tens	Ones

Most children should do this by the end of Y3

$$\begin{array}{r}
 40 \\
 50 \\
 \hline
 20
 \end{array}
 \qquad
 \begin{array}{r}
 13 \\
 3 \\
 \hline
 4
 \end{array}
 -
 \begin{array}{r}
 \\
 \\
 \hline
 29
 \end{array}
 = 29$$

Most children should do this by the end of Y2

$$74 - 47 = \boxed{}$$



Most children should do this by the end of Y3

Most children should do this by the end of Y2

Column Subtraction (With Regrouping) – Step 2

3,4

As above

As above

$$\begin{array}{r} 4 \cancel{5} 3 - \\ 24 \\ \hline \underline{29} \end{array}$$

*Most children should do this by the end of Y4
Y5 and Y6 children continue to use this method for
integers and decimal fractions.*

Multiplication

Doubling

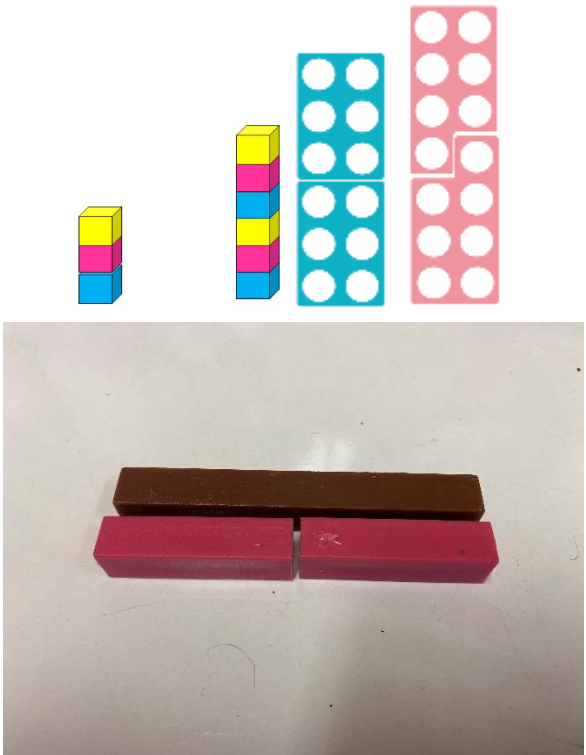
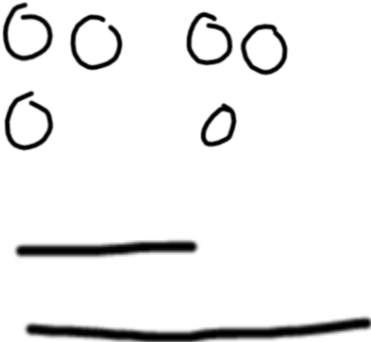
Pre-requisites and explicit pre-practice:

1-1 Correspondence

Counting of objects

Dividing objects into two equal groups

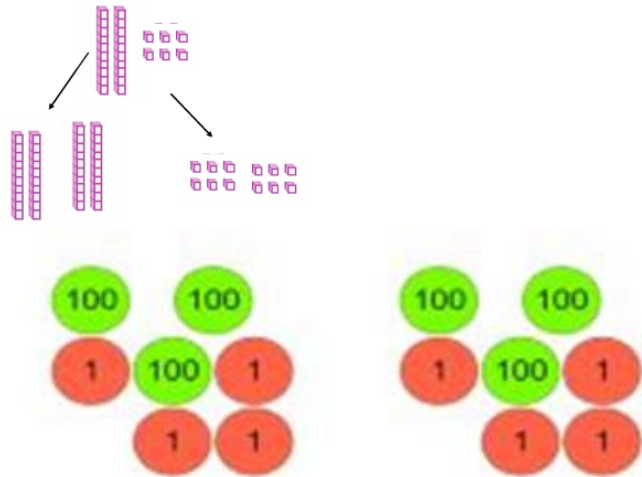
Understand that the last number counted is the amount in the set.

Main Year Groups	Concrete	Pictorial	Abstract
R, 1	 <p>The concrete section shows three stacks of blocks. The first stack has 1 block (yellow, pink, blue). The second stack has 3 blocks (yellow, pink, blue). The third stack has 6 blocks (yellow, pink, blue, yellow, pink, blue). Below this is a photograph of a brown bar and two pink bars, illustrating the doubling of a single object into two equal parts.</p>	 <p>The pictorial section shows two groups of circles. The first group has 3 circles (top row 2, bottom row 1). The second group has 3 circles (top row 2, bottom row 1). Below the circles are two horizontal lines, one short and one long, representing the doubling process.</p> <p><i>Most children should do this by the end of Y1</i></p>	$3 + 3 = 6$ <p>double? is 6</p> <p><i>Most children should do this by the end of Y1</i></p>

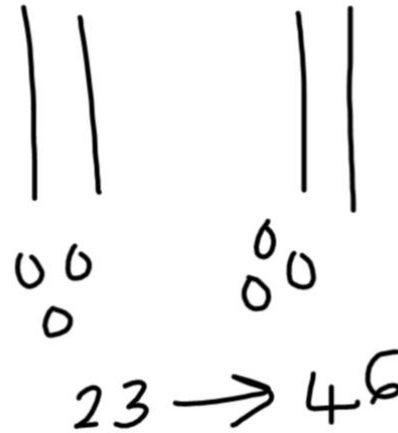
Children understand the concept of doubling by taking an amount and making it again. They should be able to explain when resources show a 'not double' and begin to explain how to make it a double.

Most children should do this by the end of R (Children are not expected to recite doubles)

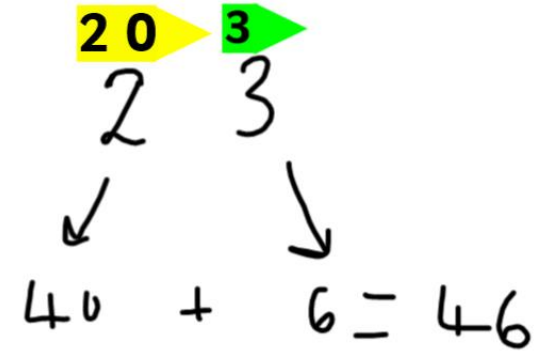
2,3



Most children should be able to do this (to the relative numbers) by the end of Y2.



Most children should be able to do this (to the relative numbers) by the end of Y2.



Most children should be able to do this (to the relative numbers) by the end of Y3 (including crossing boundaries, e.g. double 27)

Multiples, Equal Groups, Repeated Addition

Pre-requisites and explicit pre-practice:

1-1 Correspondence

Counting of objects

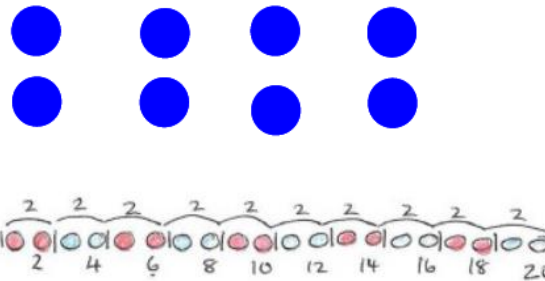
Dividing objects into two equal groups

Understand that the last number counted is the amount in the set.

R,
1



Most children should do this by the end of 1 counting in 2s, 5s and 10s.



Most children should do this by the end of Y1

2, 4, 6, 8, 10

5, 10, 15, 20, 25, 30

$$2 + 2 + 2 + 2 + 2 = 10$$

10				
2	2	2	2	2

Most children should do this by the end of Y1

Understanding Arrays

Pre-requisites and explicit pre-practice:
Spatial understanding of equal rows and columns

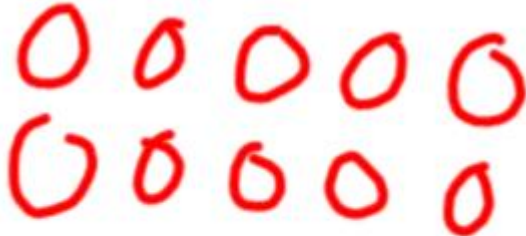
1,2,3



Most children should do this practically by the end of R



Most children should do this by the end of Y1



$$5 \times 2 = 10$$

Most children should do this by the end of Y2

Commutativity

Pre-requisites and explicit pre-practice:

Understanding of x and =

Understanding of the array

2,3

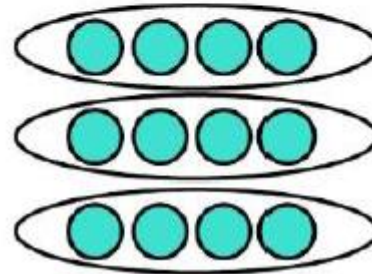
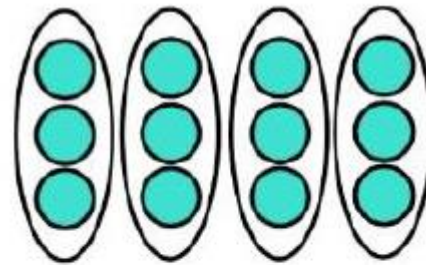
Create arrays using counters and cubes and Numicon.



Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.



Most children should do this practically by the end of Y2



Most children should do this by the end of Y2



$$5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

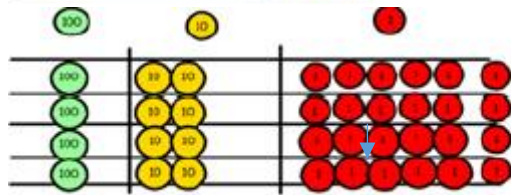
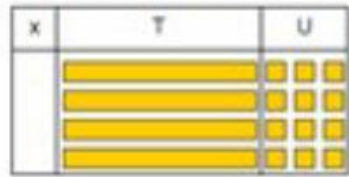
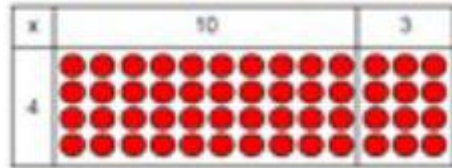
$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

Most children should do this by the end of 2

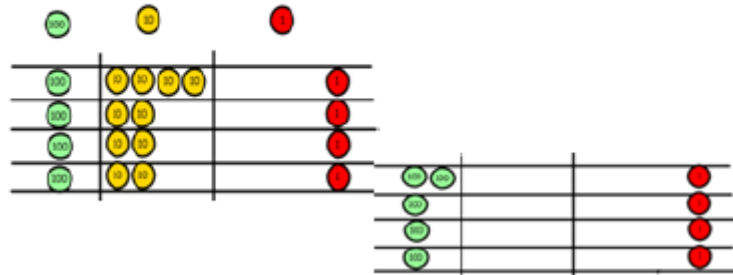
Grid Method

3,4

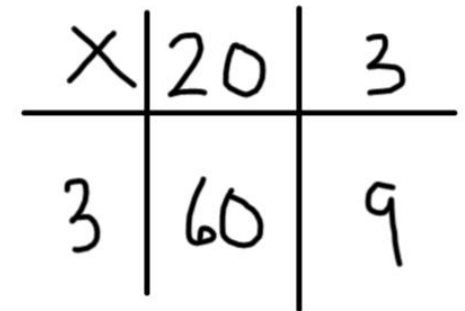
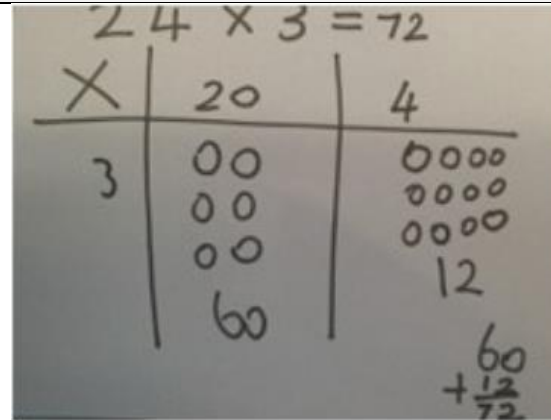


Calculations
4 x 126

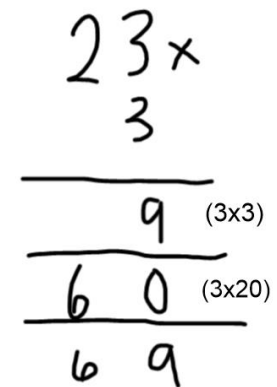
Add up each column, starting with the ones making any exchanges needed



Most children should be able to do this by the end of Y3.



Children should be able to do this by the end of Y3.



Most children should be able to do this by the end of Y4. When children are secure, they should progress on to HTO x O

Most children should be able to do this by the end of Y3.

Column Multiplication TO x TO

5, 6

$$\begin{array}{r|l}
 X & 20 \quad 4 \\
 \hline
 30 & 600 \quad 120 \\
 \hline
 6 & 120 \quad 24
 \end{array}$$

Children should be able to do this by the end of Y4.

$$\begin{array}{r}
 24x \\
 36 \\
 \hline
 24 \quad (6x4) \\
 120 \quad (6x20) \\
 \hline
 120 \quad (30x4) \\
 600 \quad (30x20) \\
 \hline
 864
 \end{array}$$

Most children should be able to do this by the end of Y5.

$$\begin{array}{r}
 24x \\
 36 \\
 \hline
 144 \\
 720 \\
 \hline
 864
 \end{array}$$

Division

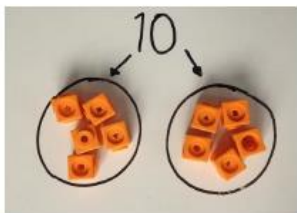
Sharing

Pre-requisites and explicit pre-practice:

Understanding of sharing equally
 Understanding of repeated addition

Main Year Groups

Concrete



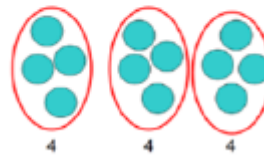
Pictorial

Children use pictures or shapes to share quantities.

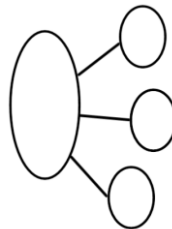


8 shared between 2 is 4

Sharing:



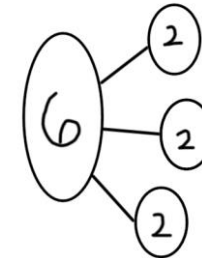
12 shared between 3 is 4



Abstract

No use of the division sign in R or Y1.

Children might write sentences: *6 sweets shared between 3 people means they each get 2 sweets.*



6		
2	2	2

By the end of Y2, most children should use and understand the division sign: $6 \div 3 = 2$

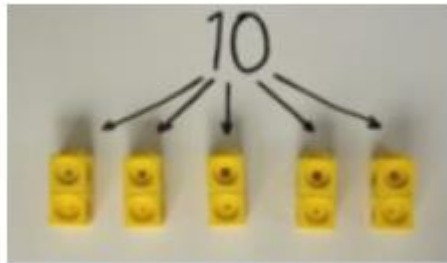
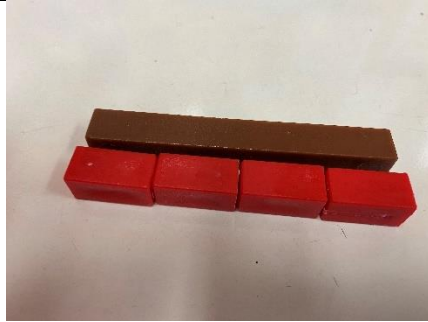
R, 1, 2, 3

Grouping

Pre-requisites and explicit pre-practice:

- Understanding of equal groups
- Skip counting for relevant multiples
- Understanding of repeated addition

R, 1, 2, 3



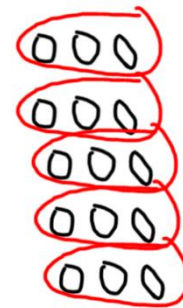
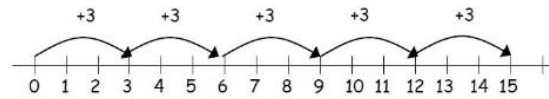
Objects should be grouped into equally sized groups in problem solving contexts and real-life contexts.



The bead string can be used for grouping.

Most children will be able to do this by the end of Y1

$$15 \div 3 = 5$$



Children build arrays by grouping into threes here.

$$15 \div 3 =$$

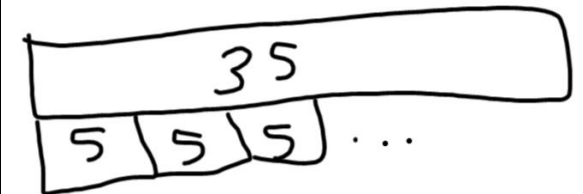
How many threes are in 15?
15 grouped into threes.

Most children will be able to do this by the end of Y2.

Children begin to count in step sizes to work out the answers to division questions.

$$35 \div 5 = 7$$

5, 10, 15, 20, 25, 30, 35



Children use the bar model to support counting in step sizes.

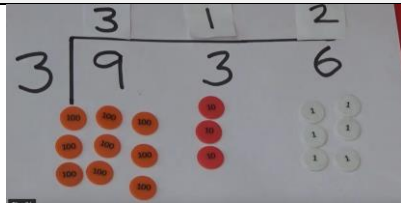
Most children will be able to do this by the end of Y2.

Short Division

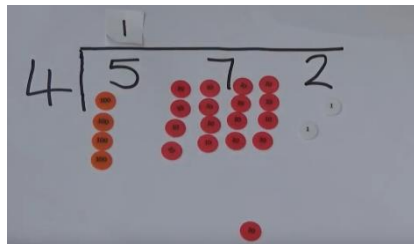
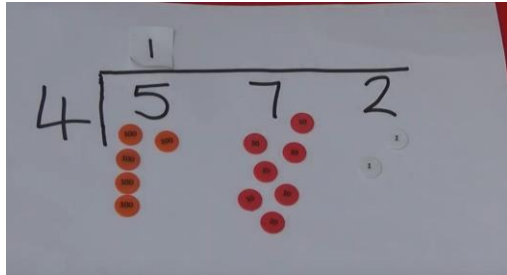
Pre-requisites and explicit pre-practice:

- Secure times tables and related division facts
- Understanding of remainders
- Understanding of value of place-value counters

4,5,6



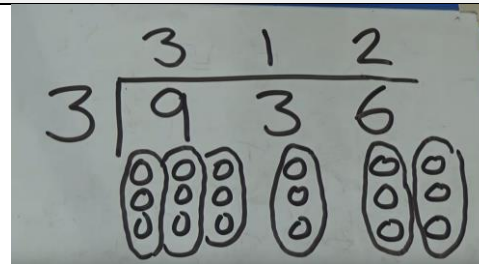
With exchanging



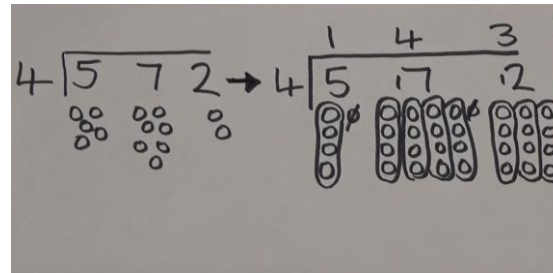
The left over 100 counter needs to be exchanged for ten 10 counters

under the 7. And so on...

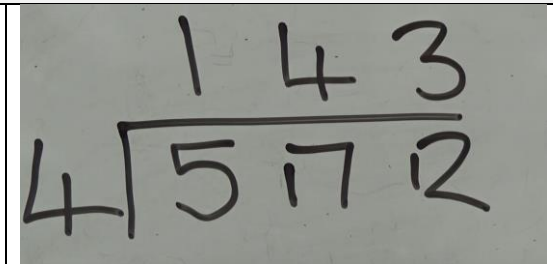
Most children should be able to do this by the end of Y4



With exchanging



Most children should be able to do this by the end of Y4



Most children should be able to do this by the end of Y4

Long Division

Pre-requisites and explicit pre-practice:

Mental addition of 2-digit numbers

Understanding of equal groups

Understanding of remainders

6

Long division - the first thing children need to do is their jottings.

Either

$$1 \times 23 = 23$$

$$2 \times 23 = 46$$

$$3 \times 23 = 69$$

$$4 \times 23 = 92$$

$$5 \times 23 = 115$$

$$6 \times 23 = 138$$

$$7 \times 23 = 161$$

$$8 \times 23 = 184$$

$$9 \times 23 = 207$$

Or

$$23$$

$$\underline{23}$$

$$46$$

$$\underline{23}$$

$$69$$

$$\underline{23}$$

$$92$$

$$\underline{23}$$

$$115$$

$$\underline{23}$$

$$138$$

$$\underline{23}$$

$$161$$

Once the children have recorded their jottings, they're ready to tackle the question.

$$1 \times 23 =$$

$$7 \times 23 =$$

$$\begin{array}{r} 107 \\ \hline 23 \overline{) 246} \\ \underline{23} \\ 16 \\ \underline{16} \\ 0 \end{array}$$

Where the two-digit number is smaller than the divisor, bring down a second number to form a three-digit number.